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BMATM201

Second Semester B.E./B.Tech. Degree Examination, Nov./Dec. 2023
Mathematics – II for Mechanical Engineering Stream

Time: 3 hrs.   Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\,dy\,dx$.	6	L3	CO1
	b.	Evaluate $\int_0^\infty \int_x^\infty \dfrac{e^{-y}}{y}\,dx\,dy$ by changing the order of the integration.	7	L3	CO1
	c.	Show that $\overline{(1/2)} = \sqrt{\pi}$.	7	L2	CO1
		OR			
Q.2	a.	Evaluate $\int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2)\,dx\,dy\,dz$.	6	L3	CO1
	b.	Evaluate $\int_{-a}^a \int_0^{\sqrt{a^2-x^2}} \left(\sqrt{x^2 + y^2}\right)dy\,dx$. By changing into polar coordinates.	7	L3	CO1
	c.	Write a mathematical tool program to find the volume of the tetrahedron bounded by the planes x = 0, y = 0 and $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$.	7	L3	CO5
		Module – 2			
Q.3	a.	Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at (2, 1, 2).	6	L2	CO2
	b.	Find the constants a, b and c such that the vector $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational.	7	L2	CO2
	c.	Find div $\vec{F}$ and curl $\vec{F}$. Where $\vec{F} = grad\ (x^3 + y^3 + z^3 - 3xyz)$ at (1, 1, 1).	7	L2	CO2
		OR			
Q.4	a.	Using Greens theorem, evaluate $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where c is the closed curve of the region bounded by $y = \sqrt{x}$ and $y = x^2$.	6	L3	CO2
	b.	Use Stokes Theorem to evaluate $\int_c \vec{F}.\vec{dr}$, where $\vec{F} = (x^2 + y^2)i - 2xyj$ and c is bounded by the lines x = ±a, y = 0, y = b.	7	L3	CO2
	c.	Write the modern mathematical tool program to find the curl of vector field. $\vec{F} = x^2yzi + y^2zxj + z^2xyk$.	7	L3	CO5
		Module – 3			
Q.5	a.	Form the partial differential equation by eliminating the arbitrary constants from the relation $(x - a)^2 + (y - b)^2 + z^2 = 4$.	6	L2	CO3

	b.	Solve $\dfrac{\partial^2 z}{\partial y^2} = z$ given that $y = 0$, $z = e^x$ and $\dfrac{\partial z}{\partial y} = e^{-x}$.	7	L3	CO3
	c.	Solve : $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ using Lagrange's multipliers.	7	L3	CO3
OR					
Q.6	a.	Form the partial differential equation by eliminating the arbitrary function from $\phi(x^2 + y^2 + z^2, xyz) = 0$.	6	L2	CO3
	b.	Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{x}{y}$, given that $\dfrac{\partial z}{\partial x} = \log x$ when $y = 1$ and $z = 0$ when $x = 1$.	7	L3	CO3
	c.	Derive one dimensional wave equation.	7	L2	CO3
Module – 4					
Q.7	a.	Find an approximate value of the root of the equation $xe^x = 2$ in the interval $(0.5, 1)$ using Regula-Falsi method correct to 4-decimal places carryout 4 iterations.	6	L3	CO4
	b.	Using Newton's backward interpolation formula find y at $x = 5.2$ for the data. 	x	1	2	3	4	5
y	10	26	58	112	194		7	L3	CO4
	c.	Evaluate $\displaystyle\int_0^1 \dfrac{1}{1 + x^2} dx$ by using Simpson's $1/3^{rd}$ rule by taking 7 ordinates.	7	L3	CO4
OR					
Q.8	a.	Find the real root of the equation $\text{Tan}x = x$ near $x = 4.5$ by Newton's – Raphson method correct to 4-decimal places.	6	L3	CO4
	b.	Using Newton's divided difference formula find $f(5)$ from the following data: 	x	0	2	3	4	7	9
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y	4	26	58	112	466	922		7	L3	CO4
	c.	Evaluate $\displaystyle\int_4^{5.2} (\log x) dx$ by using Trapezoidal rule by taking 7 ordinates.	7	L3	CO4
Module – 5					
Q.9	a.	Using Taylors series method to solve $\dfrac{dy}{dx} = (x + y)$ with $y(1) = 0$ at the points $x = 1.1$ and 1.2 upto 4th approximation.	6	L2	CO4
	b.	Using Runge-Kutta method of fourth order find $y(0.2)$ given that $\dfrac{dy}{dx} = (x + y^2); y(0) = 1$ Taking $h = 0.2$.	7	L3	CO4
	c.	Using modified Euler's method find y at $x = 0.2$ given that $\dfrac{dy}{dx} = 3x + \dfrac{y}{2}$; $y(0) = 1$ taking $h = 0.1$. Carryout 3-modification in each step.	7	L3	CO4
OR					
Q.10	a.	Using Runge-Kutta method of fourth order find $y(0.1)$ given that $\dfrac{dy}{dx} = 3e^x + 2y$, with $y(0) = 0$ taking $h = 0.1$.	6	L3	CO4
	b.	Given $\dfrac{dy}{dx} = (x - y^2)$ and the data $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ compute $y(0.8)$ by Milne's method.	7	L3	CO4
	c.	Using mathematical tools write a code to find the solution of $\dfrac{dy}{dx} = \left(1 + \dfrac{y}{x}\right)$ at $y(2)$ taking $h = 0.2$ given that $y(1) = 2$ by Runge-Kutta method.	7	L3	CO5